Reduction of Image Quality Degradation of JPEG Decoded Image 
based on Image Restoration Method

Kazuhiro FUJITA, Kazuya IMAFU, and Yasuo YOSHIDA

Department of Electronics and Information Science,
Kyoto Institute of Technology
Matsugasaki, Sakyo-ku, Kyoto 606-8585, JAPAN
Email: fujita@d.kit.ac.jp

Abstract

JPEG image is degraded by blocking noise and mosquito noise caused by quantization on Block DCT (Discrete Cosine Transform) domain. The purpose of this paper is reduction of image quality degradation by means of the image restoration method instead of the simple decoding method.

1. Introduction

We see the following image quality degradation according to the Block DCT based compression in JPEG image.

• Blocking Artifacts
• Mosquito Noise

In this paper, our purpose is to reduce the above image quality degradation based on image restoration method instead of the simple decoding method. Our proposed method is based on the edge-adaptive restoration method; the regularizing operator is depend on the edge orientation, and the regularizing parameter is depend on the local activity.

2. Coding and Decoding of JPEG Image

In this paper, a $M \times N$ image is treated as a vector, in which the $MN$ pixels are lexicographically ordered. The coded image $\tilde{z}_2$ of the original image $\tilde{x}$ is given by the following equation.

$$\tilde{z}_2 = QC[\tilde{x}]$$

where $C$ denotes Block DCT with size $MN \times MN$, in which the block size of the Block DCT is $8 \times 8$, $Q$ denotes the quantizing operator.

The decoded image $\hat{x}$ of the coded image $\tilde{z}_2$ is given by the following equation.

$$\hat{x} = C^{-1}Q^{-1}[\tilde{z}_2]$$

where $C^{-1}$ denotes inverse Block DCT with size $MN \times MN$, in which the block size of the inverse Block DCT is $8 \times 8$, $Q^{-1}$ denotes the dequantizing operator.

3. Image Model

3.1. Model of Original Image

The following white noise driven image model is used for the original image.

$$A \tilde{x} = \tilde{w}$$

where $\tilde{x}$ denotes the original image with size $MN$, $A$ denotes whitening filter of the original image with size $MN \times MN$, and $\tilde{w}$ denotes the driving white noise with the average 0, whose variance is assumed to be space variant according to the position in the image. The auto-covariance matrix of the original image $\tilde{x}$ is defined by

$$R_w = E[\tilde{w}\tilde{w}^T]$$

$$= \text{Diag}(\ldots, \sigma^2_{\tilde{x}}(m,n), \ldots)$$

The scalar equation corresponding to Eq.(3) is as follows

$$x_{m,n} = \sum_{(k,l) \in S} a_{k,l}(\theta(m,n)) x_{m-k,n-l} + w_{m,n}$$

where $\theta(m,n)$ denotes the edge orientation which is defined by the following equation.

$$\theta(m,n) = \begin{cases} 
0^\circ & \text{edge region with } 0^\circ \\
45^\circ & \text{edge region with } 45^\circ \\
90^\circ & \text{edge region with } 90^\circ \\
135^\circ & \text{edge region with } 135^\circ \\
360^\circ & \text{flat region}
\end{cases}$$

where $360^\circ$ means the flat region for the convenience. The coefficients of the whitening filter $a_{k,l}(\theta)$ is shown in Table1.
### 3.2. Degradation Model

The process of coding and decoding make the original image to be degraded. Its degradation is assumed to be modeled by the following equation.

\[
\tilde{x} \approx C^{-1}(Q^{-1}[\tilde{z}] + \tilde{v})
\]

where \( \tilde{v} \) denotes quantization noise with the auto-covariance matrix \( R_v \):

\[
R_v = E[\tilde{v}\tilde{v}^T]
\]

The Eq. (8) is modified to the following equation in the style of the well-known degradation equation.

\[
Q^{-1}[\tilde{z}] = C\tilde{x} - \tilde{v}
\]

The above equation shows that the original image \( \tilde{x} \) is degraded by Block DCT \( C \) and the addition of the noise \( -\tilde{v} \), then the observed image \( Q^{-1}[\tilde{z}] \) is obtained.

### 4. Reduction of Image Quality Degradation of JPEG Image

#### 4.1. Algorithm of Reduction of Image Quality Degradation

1. The decoded image \( \hat{x} \) is obtained by the simple JPEG decoding.
2. The edge orientation \( \theta(m, n) \) is estimated from \( \hat{x} \).
3. The auto-covariance matrix \( R_u \) of the original image is estimated from the output \( Ax \) of the whitening filter.
4. The improved image \( \hat{x} \) is obtained by minimizing the estimate \( J(\hat{x}) \) of the improved image \( \hat{x} \).

#### 4.2. Detection of Edge Orientation

The edge orientation is extracted by the following steps.

**Compass operator** The compass operator is applied to the simply decoded JPEG image \( \hat{x} \).

\[
\phi_{m,n} = \arg_{\theta \in \Theta} \max
\]

### 4.3. Estimation of the auto-covariance matrix \( R_u \) of driving white noise

The element \( \sigma_{m,n}^2 \) of the auto-covariance matrix \( R_u \) of the driving white noise \( w_{m,n} \) is estimated by averaging the output of the whitening filter \( A \) over the window \((2W+1) \times (2W+1) \).

\[
w_{m,n} = \frac{1}{(2W+1)^2} \sum_{k=-W}^{W} \sum_{l=-W}^{W} w_{m,n}
\]

### 4.4. Evaluation of Improved Image

The improved image \( \hat{x} \) is evaluated by the following equation.

\[
J(\hat{x}) = (Q^{-1}[\tilde{z}] - C\hat{x})^T R^{-1}_u (Q^{-1}[\tilde{z}] - C\hat{x})
\]

\[+ (A\hat{x})^T R^{-1}_w (A\hat{x})\]

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**Table 1: Whitening Filter**

<table>
<thead>
<tr>
<th>( \theta = 0^\circ )</th>
<th>( \theta = 45^\circ )</th>
<th>( \theta = 90^\circ )</th>
<th>( \theta = 135^\circ )</th>
<th>( \theta = 360^\circ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_{1,1}(\theta) )</td>
<td>( a_{1,0}(\theta) )</td>
<td>( a_{0,1}(\theta) )</td>
<td>( a_{0,0}(\theta) )</td>
<td>( a_{0,-1}(\theta) )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2\alpha</td>
<td>0</td>
</tr>
<tr>
<td>( \alpha_{-1,0}(\theta) )</td>
<td>( \alpha_{0,-1}(\theta) )</td>
<td>( \alpha_{1,-1}(\theta) )</td>
<td>( \alpha_{0,-1,0}(\theta) )</td>
<td>( \alpha_{0,-1,1}(\theta) )</td>
</tr>
<tr>
<td>2\alpha</td>
<td>0</td>
<td>2\alpha</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

\[
t_{m,n} = \left| \sum_{k=-1}^{1} \sum_{l=-1}^{1} c_{k,l}(\theta) \tilde{x}_{m-k,n-l} \right|
\]

\[
\text{Thresholding} \quad \theta(m,n) = \begin{cases} \phi_{m,n}, & t_{m,n} \geq T \\ 360^\circ, & t_{m,n} < T \end{cases}
\]
Table 2: Compass Operator

<table>
<thead>
<tr>
<th>$c_{1,1}(	heta)$</th>
<th>$c_{1,0}(	heta)$</th>
<th>$c_{1,-1}(	heta)$</th>
<th>$\theta = 0^\circ$</th>
<th>$\theta = 45^\circ$</th>
<th>$\theta = 90^\circ$</th>
<th>$\theta = 135^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-1</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
</tr>
</tbody>
</table>

The above equation is modified to the following equation.

$$J(\hat{x}) = \{C(\hat{x} - \hat{\hat{x}})\}^T R_v^{-1} \{C(\hat{x} - \hat{\hat{x}})\} + (A\hat{x})^T R_w^{-1} (A\hat{x})$$

(17)

This equation shows that the improved image $\hat{x}$ is evaluated by calculating the error between the simply decoded image $\hat{x}$ and the improved image $\hat{x}$ on Block DCT domain with the weight of the variance $R_v$ of the quantization noise.

The steepest descendent method is applied to the evaluating function of improved images ( Eq. 17 ) in order to derive the iterative processing method. The derived iterative processing method is as follows:

1. Initial value of the improved image

$$\hat{x}^{(0)} = \hat{x}$$

(18)

2. Initial values of the parameters

$$k = 0$$

(19)

$$\beta^{(0)} = 0$$

(20)

3. Evaluation of $\nabla J$

$$\hat{g}^{(k)} = \nabla J(\hat{x}^{(k)}) = -2 \{C^T R_v^{-1} C(\hat{x} - \hat{x}^{(k)}) - A^T R_w^{-1} A\hat{x}^{(k)}\}$$

(22)

4. Convergence criterion test

$$IF \| \hat{g}^{(k)} \|^2 < \epsilon \ THEN \ END$$

(23)

5. Evaluation of search direction

$$\hat{p}^{(k)} = \begin{cases} -\hat{g}^{(0)} & k = 0 \\ -\hat{g}^{(k)} + \beta^{(k)} \hat{p}^{(k-1)} & k \geq 0 \end{cases}$$

(24)

$$\beta^{(k)} = \frac{\| \hat{g}^{(k)} \|^2}{\| \hat{g}^{(k-1)} \|^2}$$

(25)

6. Update of the improved image $\hat{x}^{(k)}$

$$\hat{x}^{(k+1)} = \hat{x}^{(k)} + \gamma^{(k)} \hat{p}^{(k)}$$

(26)

$$\gamma^{(k)} = \arg \min_{\gamma} J(\hat{x}^{(k)} + \gamma \hat{p}^{(k)})$$

(27)

$$= -\frac{1}{2} \frac{\hat{g}^{(k)^T} \hat{p}^{(k)}}{\| \hat{g}^{(k)} \|^2}$$

$$/ \{C \hat{p}^{(k)} \}^T R_v^{-1} (C \hat{p}^{(k)}) + (A \hat{p}^{(k)})^T R_w^{-1} (A \hat{p}^{(k)})$$

(28)

5. Computer Experiment

5.1. Estimate of Images

The following PSNR ( Peak-Signal-to-Noise-Ratio ) is used to evaluate the distance between an image $\hat{f}$ and an image $\hat{g}$.

$$PSNR[\text{dB}] = 10 \log_{10} \frac{255^2}{M \times N \| \hat{f} - \hat{g} \|^2}$$

(29)

where 255 is the maximum gray level.

5.2. Computer Simulation

The results of our computer simulation is shown in Fig 1, Fig 2 and Table 3, where PSNR between the original image and improved image / JPEG image is shown.

Fig 2 (a) is original image, Fig 2 (b) is the simply decoded JPEG image, and Fig 2 (c) is the improved image which is processed by our proposed method. Fig 2 (d) is the squared error image between (b) and (a), and Fig 2 (e) is the squared error image between (c) and (a). Fig 2 (f), (h) show that the blocking artifact and mosquito noise have been reduced by our proposed method. Fig 1, Table 3 shows that our proposed method is effective.

6. Conclusions

The results of our computer simulation show that the blocking noise and mosquito noise are reduced by our proposed method, therefore the effectiveness of our proposed method is shown.
Figure 1: Rate-PSNR

Table 3: PSNR on Computer Simulation

<table>
<thead>
<tr>
<th>bit/pixel</th>
<th>JPEG Image</th>
<th>Improved Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.96</td>
<td>45.80</td>
<td>46.08</td>
</tr>
<tr>
<td>1.51</td>
<td>45.30</td>
<td>45.50</td>
</tr>
<tr>
<td>1.00</td>
<td>44.61</td>
<td>44.76</td>
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<tr>
<td>0.90</td>
<td>44.45</td>
<td>44.59</td>
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<tr>
<td>0.81</td>
<td>44.29</td>
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</tr>
<tr>
<td>0.70</td>
<td>44.03</td>
<td>44.17</td>
</tr>
<tr>
<td>0.60</td>
<td>43.72</td>
<td>43.87</td>
</tr>
<tr>
<td>0.50</td>
<td>43.33</td>
<td>43.49</td>
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<tr>
<td>0.40</td>
<td>42.70</td>
<td>42.88</td>
</tr>
<tr>
<td>0.31</td>
<td>41.71</td>
<td>41.96</td>
</tr>
</tbody>
</table>

Figure 2: Results of Computer Simulation