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Evaluation of Blurred Image and Restored Image based on Characteristics of the Spectrum of Output of MDF

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Abstract

We propose the method of estimating the parameters of the original image, and the evaluating blurred image as well as the restored image based on the characteristics of the spectrum of the output of MDF. MDF (Measurement Differencing Filter) is defined by as the inverse filter of the shaping filter about the stochastic original image model.

1. Introduction

We show the method of estimating the parameters of the non-causal linear prediction model driven by white noise about the original image. This method based on that when MDF is equal to the frequency response of the whitening filter about the original image, the output of the MDF is white. We estimate the parameters by minimizing the skewness and the flatness of the spectrum of the output of MDF.

We propose the method of evaluating the blurness of the blurred image. The blurness is evaluated by the skewness and the flatness of the spectrum of the output of the MDF. This method can be applied to judging whether the object is in-focus or not.

We propose the method of evaluating the restored image in order to judge whether the restoring process is valid or not. The restored image is evaluated by the skewness and the flatness of the spectrum of the output of the MDF. This method can be applied to estimating the parameters of the blurred image, and adjusting the parameters on image restoration.

2. Image Model

The following linear prediction model driven by white noise is applied to the original image $x_{m,n}$.

$$x_{m,n} = \sum_{(k,l)\in S} a_{k,l} x_{m-k,n-l} + u_{m,n} \qquad (1)$$

where $a_{k,l}$ denotes the coefficient of linear prediction, $u_{m,n}$ denotes the driving white noise with average 0, variance σ_u^2.

The Spectrum of the original image is given by the following equations.

$$P_x(\omega_1,\omega_2) = \frac{\sigma_u^2}{|A(e^{-i\omega_1}, e^{-i\omega_2})|^2} \qquad (2)$$

$$A(Z_1^{-1}, Z_2^{-1}) = 1 - \sum_{(k,l)\in S_a} a_{k,l} Z_1^{-k} Z_2^{-l} \qquad (3)$$

When an isotropic nearest neighbor model is assumed, the original model equation is given by

$$x_{m,n} = \alpha(x_{m+1,n} + x_{m-1,n} + x_{m,n+1} + x_{m,n-1}) + u_{m,n} \qquad (4)$$

The blur process is modeled by the following equation.

$$y_{m,n} = \sum_{(k,l)\in S_b} b_{k,l} x_{m-k,n-l} + v_{m,n} \qquad (5)$$

where $b_{k,l}$ denotes the coefficient of the blur system, $v_{m,n}$ denotes the additive white noise with average 0, variance σ_v^2. The spectrum of the blurred image is given by the following equations.

$$P_y(\omega_1,\omega_2) = \sigma_u^2 \frac{|B(e^{-i\omega_1}, e^{-i\omega_2})|^2}{|A(e^{-i\omega_1}, e^{-i\omega_2})|^2} + \sigma_v^2 \qquad (6)$$

$$B(Z_1^{-1}, Z_2^{-1}) = \sum_{(k,l)\in S_b} b_{k,l} Z_1^{-k} Z_2^{-l} \qquad (7)$$

When the blur process is the out-of-focus blur, the coefficient of the blur system is given by the following equations.

$$b_{k,l} = \int_{k-\frac{1}{2}}^{k+\frac{1}{2}} \int_{l-\frac{1}{2}}^{l+\frac{1}{2}} b(x,y)\,dx\,dy \qquad (8)$$

$$b(x,y) = \begin{cases} \frac{1}{\pi R^2}, & \sqrt{x^2+y^2} \le R \\ 0, & \sqrt{x^2+y^2} > R \end{cases} \qquad (9)$$

where R is the radius of the out-of-focus blur.

When the isotropic nearest neighbor image model as the original image model, and the out-focus-blur as the blur process are used, the parameters θ which is necessary for restoration are $\{\alpha, \sigma_u^2, R, \sigma_v^2\}$.

3. MDF (Measurement Differencing Filter)

The output $e_{m,n}$ of MDF to the input image $f_{m,n}$ is defined by

$$e_{m,n} = f_{m,n} - \sum_{(k,l)\in S_d} d_{k,l} f_{m-k,n-l} \tag{10}$$

The spectrum $P_e(\omega_1, \omega_2)$ of the output $e_{m,n}$ is given by

$$P_e(\omega_1, \omega_2) = |D(e^{-i\omega_1}, e^{-i\omega_2})|^2 P_f(\omega_1, \omega_2) \tag{11}$$

$$D(Z_1^{-1}, Z_2^{-1}) = 1 - \sum_{(k,l)\in S_d} d_{k,l} Z_1^{-k} Z_2^{-l} \tag{12}$$

4. Skewness and Flatness of the Spectrum

The skewness S_x,S_y and the flatness F_x,F_y of the spectrum are defined by the following equations

$$F_x = \frac{1}{\sigma_e^4(\pi - \omega_0)} \int_{\omega_0}^{\pi} \{P_e(\omega_1, 0) - \mu_e\}^4 d\omega_1 \tag{13}$$

$$S_x = \left| \frac{1}{\sigma_e^4(\pi - \omega_0)} \int_{\omega_0}^{\pi} \{P_e(\omega_1, 0) - \mu_e\}^3 d\omega_1 \right| \tag{14}$$

$$\sigma_e^2 = \frac{1}{\pi - \omega_0} \int_{\omega_0}^{\pi} \{P_e(\omega_1, 0) - \mu_e\}^2 d\omega_1 \tag{15}$$

$$\mu_e = \frac{1}{\pi - \omega_0} \int_{\omega_0}^{\pi} P_e(\omega_1, 0) d\omega_1 \tag{16}$$

F_y,S_y are defined equally.

5. Parameter Estimation of Original Image

The output $e_{m,n}$ of the MDF for the original image $x_{m,n}$ is defined by

$$e_{m,n} = D(Z_1^{-1}, Z_2^{-1}) x_{m,n} \tag{17}$$

The spectrum $P_e(\omega_1, \omega_2)$ is given by

$$P_e(\omega_1, \omega_2) = \sigma_u^2 \frac{|D(e^{-i\omega_1}, e^{-i\omega_2})|^2}{|A(e^{-i\omega_1}, e^{-i\omega_2})|^2} \tag{18}$$

When the following equation is assumed,

$$|A(e^{-i\omega_1}, e^{-i\omega_2})|^2 \simeq |D(e^{-i\omega_1}, e^{-i\omega_2})|^2 \tag{19}$$

then the spectrum $P_e(\omega_1, \omega_2)$ becomes

$$P_e(\omega_1, \omega_2) \simeq \sigma_u^2 \tag{20}$$

i.e. the spectrum is white, and constant. Here, we define the estimate as the product of the skewness S_x,S_y and the flatness F_x,F_y of the spectrum $P_e(\omega_1, \omega_2)$;

$$L_{\text{ori}}(\{d_{k,}\}) = S_x\, S_y\, F_x\, F_y \tag{21}$$

The parameter estimation is performed by minimizing the estimate L_{ori}.

In our computer simulation, an isotropic nearest neighbor image model is used as the original image model, then $A(Z_1^{-1}, Z_2^{-1})$,$D(Z_1^{-1}, Z_2^{-1})$ are given by

$$A(Z_1^{-1}, Z_2^{-1}) = 1 - \alpha(Z_1^{-1} + Z_1 + Z_2^{-1} + Z_2) \tag{22}$$

$$D(Z_1^{-1}, Z_2^{-1}) = 1 - \tilde{\alpha}(Z_1^{-1} + Z_1 + Z_2^{-1} + Z_2) \tag{23}$$

The estimate $L_{\text{ori}}(\tilde{\alpha})$ to the original image (Fig. 1) generated with $\alpha = 0.248$ is shown in fig2. In fig2, when $\tilde{\alpha}$ is 0.2474, L_{ori} is smallest. Therefore the estimated value $\hat{\alpha}$ is 0.2474.

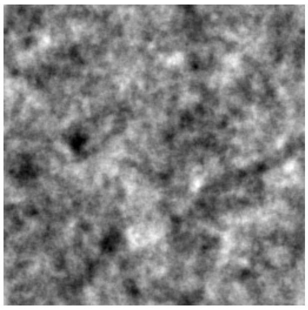

Figure 1: Generated Image $\alpha = 0.248$

6. Evaluation of Blurred Image

The output $e_{m,n}$ of the MDF for the blurred image $y_{m,n}$ is defined by

$$e_{m,n} = D(Z_1^{-1}, Z_2^{-1}) y_{m,n} \tag{24}$$

When Eq.(19) is assumed, then

$$P_e(\omega_1, \omega_2) = \sigma_u^2 |B(e^{-i\omega_1}, e^{-i\omega_2})|^2 + \sigma_v^2 |D(e^{-i\omega_1}, e^{-i\omega_2})|^2 \tag{25}$$

We define $\tilde{P}_e(\omega_1, \omega_2)$;

$$\tilde{P}_e(\omega_1, \omega_2) = \max(P_e(\omega_1, \omega_2) - \sigma_v^2 |D(e^{-i\omega_1}, e^{-i\omega_2})|^2, \epsilon) \tag{26}$$

Here, we define the estimate of the blurred image as the product of the S_x,S_y and the flatness F_x,F_y of the spectrum $\tilde{P}_e(\omega_1, \omega_2)$;

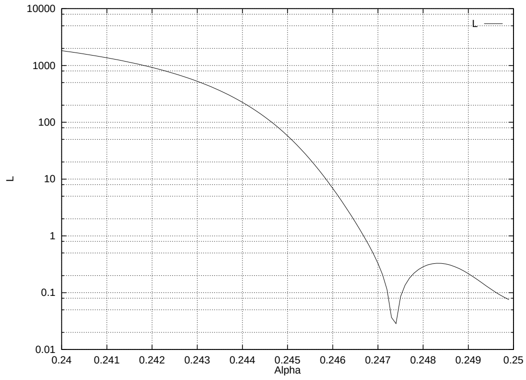

Figure 2: Estimation of α ($\tilde{\alpha}$-L_{ori})

$$L_{\text{blur}} = S_x\, S_y\, F_x\, F_y \tag{27}$$

In our computer simulation, we assume the following equation.

$$D(Z_1^{-1}, Z_2^{-1}) = 1 - \frac{1}{4}(Z_1^{-1} + Z_1 + Z_2^{-1} + Z_2) \tag{28}$$

The estimate L_{blur} to the blurred images (e.g. Fig. 3 (a),(b),(c)) that are generated from the original image forecast with $R = 1.5, 2.0, 2.5$ respectively, and $BSNR = 60[dB]$ is shown in Fig4. We can see that the relationship between the out-of-focus radius R and $\log L_{\text{blur}}$ is almost linear. Therefore, the L_{blur} is able to be used for evaluating the blurness of the blurred image.

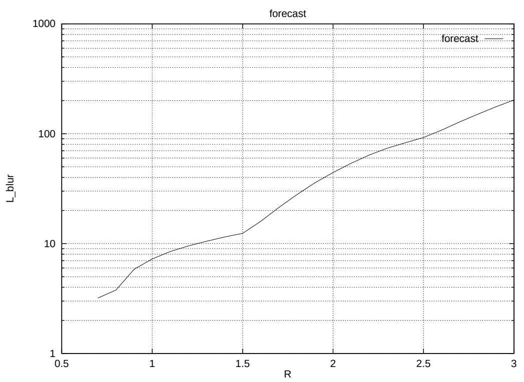

Figure 4: Estimate of the Blurred ImageL_{blur}

7. Evaluation of Restored Image

The output $e_{m,n}$ of the MDF for the restored image $\hat{x}_{m,n}$ is defined by

$$e_{m,n} = D(Z_1^{-1}, Z_2^{-1})\hat{x}_{m,n} \tag{29}$$

Here we define the estimate of the restored image as the product of the skewness S_x and the flatness F_x of $P_e(\omega_1, \omega_2)$;

$$L_{\text{res}} = S_x\ F_x \tag{30}$$

In our computer simulation, we assume Eq.(28) as the MDF. The estimate L_{res} for the images restored by an edge-adaptive iterative method from the image blurred with $R = 2.5$, $BSNR = 30[dB]$ is shown in Fig6. The MSE (Mean Square Error) between the original image (ITE-forecast) and the restored image is also shown in Fig6. We can find that the estimate L_{res} is similar to the MSE. Therefore, the estimate L_{res} is able to be used to evaluating the restored image.

If the out-of-focus radius R used by restoring process is less than the true out-of-focus radius, the restored image is not so clear and the estimate L_{res} is not small, because the blurness is still remained. If the out-of-focus radius R used by restoring process is greater than the true out-of-focus radius, the restored image is not so good and the estimate L_{res} is not small, because there are many strong ringing artifacts in the restored image.

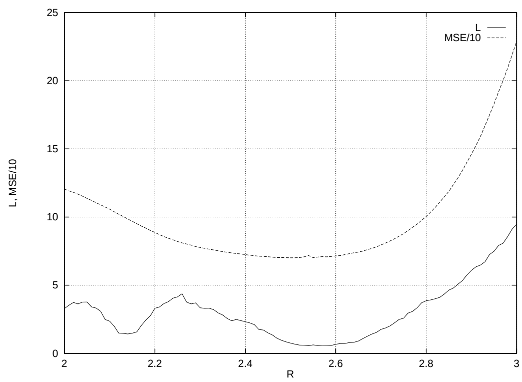

Figure 6: Estimate of the Restored ImageL_{res}

8. Conclusions

In this paper, the methods of evaluating both the blurred image and the restored image are proposed. It is shown that logarithm of the proposed estimate for the blurred image is almost linear to the radius of the out-of-focus. And it is shown that the proposed estimate for the restored image is similar to the MSE (Mean Square Error) between the restored image and the original image.

(a)Blurred Image ($R = 1.5$)
$L_{\text{blur}} = 1.88$

(b)Blurred Image ($R = 2.0$)
$L_{\text{blur}} = 8.64$

(c)Blurred Image ($R = 2.5$)
$L_{\text{blur}} = 23.74$

Figure 3: Blurred Images

(a)Original Image

(b)Blurred Image ($R = 2.5$)

(c)Restored Image ($R = 2.0$)
$L_{\text{res}} = 2.82$
$MSE = 120.53$

(d)Restored Image ($R = 2.5$)
$L_{\text{res}} = 1.10$
$MSE = 70.09$

(e)Restored Image ($R = 3.0$)
$L_{\text{res}} = 9.18$
$MSE = 228.85$

Figure 5: Restored Images